Name ____________________ Period ________

6.1A Graphing Cubic Functions: Significant Features

1. Enter the equation $y = x^3 - x^2 - 12x$ into a graphing calculator and use a table of values to draw the graph.

a) Over what interval(s) is the graph increasing?

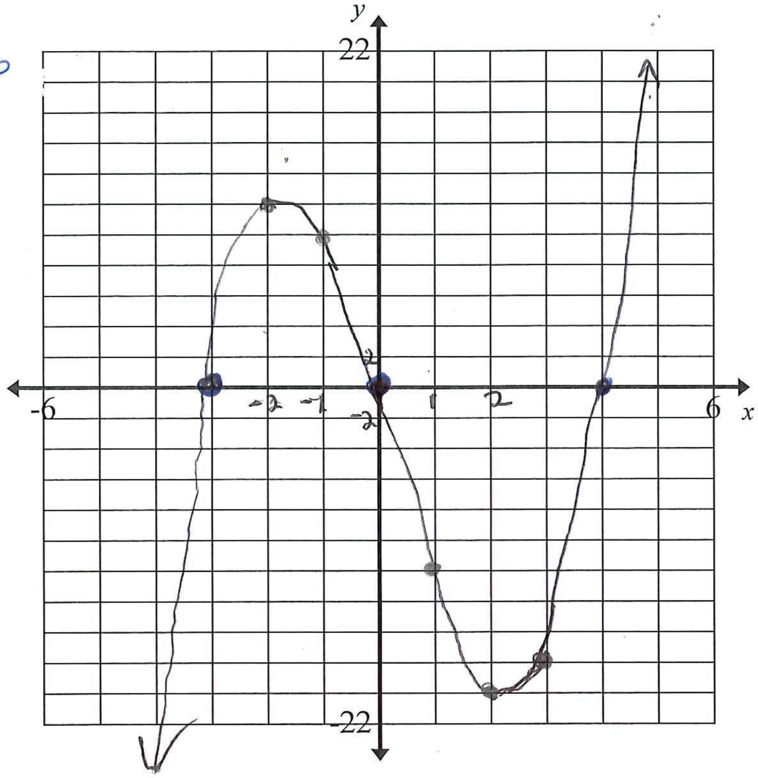
$x < -1.69$ & $x > 2.36$
(graphing calc)

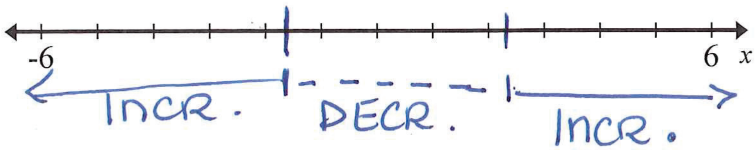
b) Over what interval(s) is the graph decreasing?

$-1.69 < x < 2.36$
(graphing calc)

c) Identify any maximum or minimum values, recording what are they and where they occur?

rel max $(-1.69, 12.56)$
rel min $(2.36, -20.75)$
These occur at turning points.





d) Identify the x-intercept(s).

$(-3, 0)$
$(0, 0)$
$(4, 0)$

e) Identify the y-intercept(s).

$(0, 0)$

f) Are there any restrictions on the domain and range? If yes, what are they?

No

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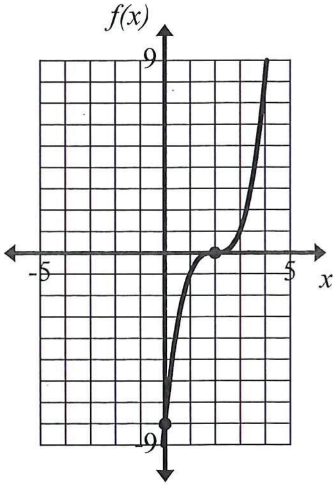
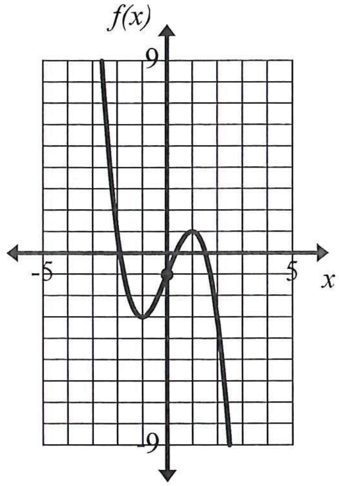
6.1A *Graphing Cubic Functions: Significant Features*

2. A *cubic function* is described by $f(x) = ax^3 + bx^2 + cx + d$, where *a, b, c,* and *d* are the coefficients and $a \neq 0$. Graphs of cubic functions show a bit more variety than those for linear or quadratic functions. Here are some examples of cubic functions with their graphs:

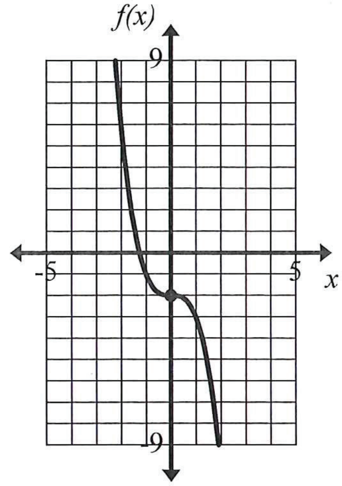
[A] $f(x) = x^3 - 6x^2 + 12x - 8$



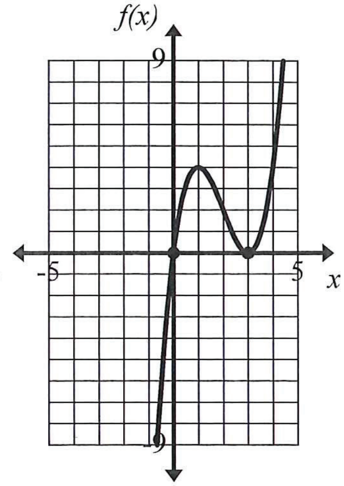
[B] $f(x) = -x^3 + 3x - 1$



[C] $f(x) = -x^3 - 2$



[D] $f(x) = x^3 - 6x^2 + 9x$



a) Identify the lead coefficient of each cubic function.

Function **[A]**: 1

Function **[B]**: −1

Function **[C]**: −1

Function **[D]**: 1

b) Make a conjecture about what influence the **sign** of the lead coefficient, *a,* has on the shape of the graph.

If a > 0, the branches that tend to infinity both have a positive slope. ↙↗

If a < 0, " " " " " " " " " negative slope. ↖↘

c) How would you describe the different types of cubic graphs? What do they have in common? How are they different?

They look like elongated "S" or "Z" curves rotated 90°

Common feature: The ends of the S or Z curve tend to ± ∞.

Different feature: The # of times the graph crosses the x-axis, either 1, 2, or 3 times.

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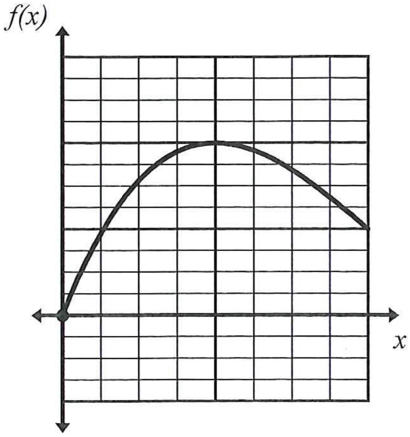
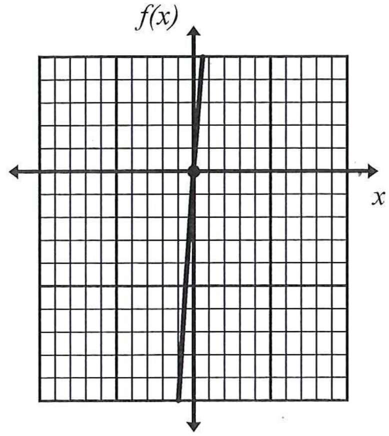
6.1A *Graphing Cubic Functions: Significant Features*

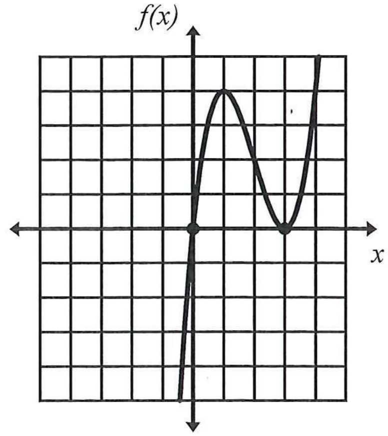
3. Kerry, Johanna, and Meng all used their graphing calculators to graph the function $f(x) = x^3 - 6x^2 + 9x$. They all entered the correct equation.



Kerry



Johanna



Meng

a) Explain why they got such different graphs for the same function.

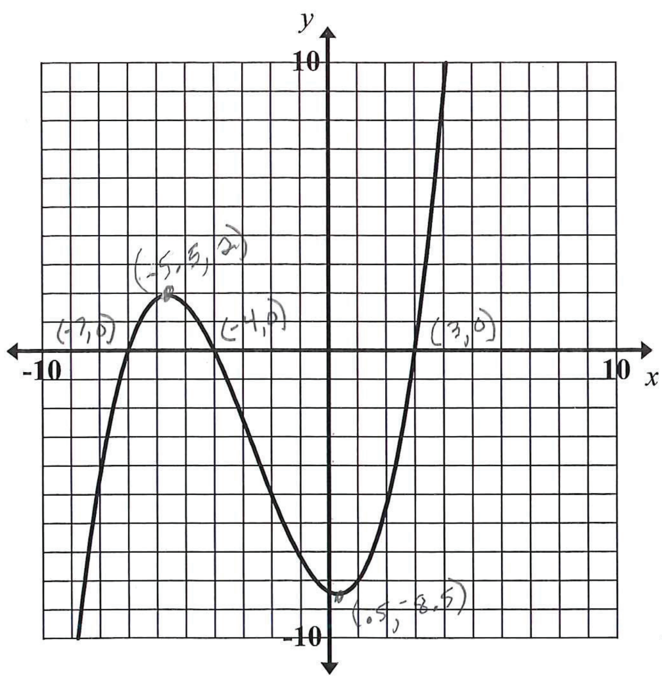
Different windows for x min, x max, y min, y max

b) Whose graph is the best representation of the cubic function? Why?

Meng, since you can see all the zeros, max & min

#4 – 6: For each graph, identify the significant features of the graph.

4.



Sign of the Lead Coefficient: Positive

Domain: all reals

Range: all reals

relative minimum: (0.5, -8.5)

relative maximum: (-5.5, 2)

interval(s) where functions values are increasing:

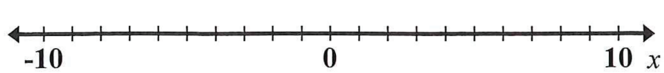
$X < -5.5$ and $X > 0.5$

interval(s) where functions values are decreasing:

$-5.5 < X < 0.5$

x–intercept(s): (-7,0), (-4,0), (3,0)

y-intercept: ≈ $(0, -8.\overline{3})$



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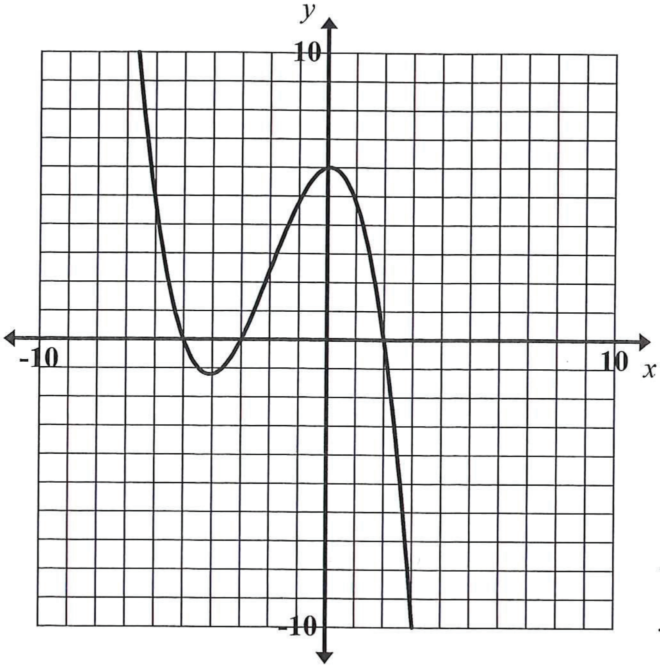
6.1A *Graphing Cubic Functions: Significant Features*

#4 – 6 (continued): For each graph, identify the significant features of the graph.

5.



Sign of the Lead Coefficient: negative

Domain: all reals

Range: all reals

relative minimum: (-4, -1.2*) approx. y value

relative maximum: (0, 6)

interval(s) where functions values are increasing:

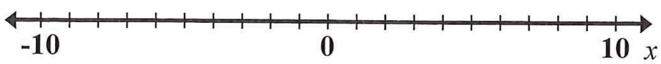
$-4 < x < 0$

interval(s) where functions values are decreasing:

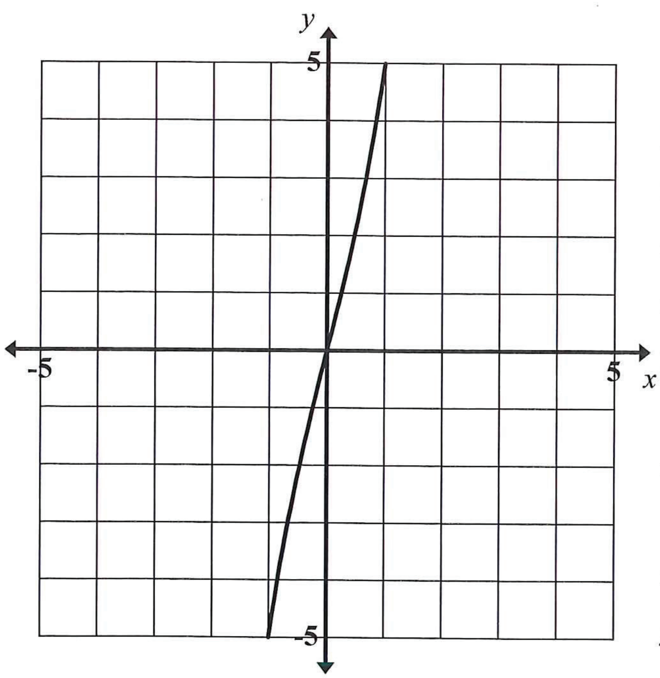
$x < -4$ and $x > 0$

x–intercept(s): (-5, 0), (-3, 0), (2, 0)

y-intercept: (0, 6)



6.



Sign of the Lead Coefficient: positive

Domain: all reals

Range: all reals

relative minimum: none

relative maximum: none

interval(s) where functions values are increasing:

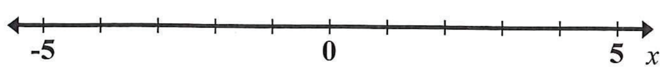
$-\infty < x < \infty$

interval(s) where functions values are decreasing:

none

x–intercept(s): (0, 0)

y-intercept: (0, 0)

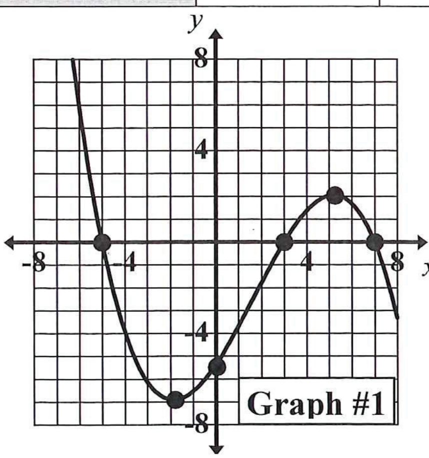


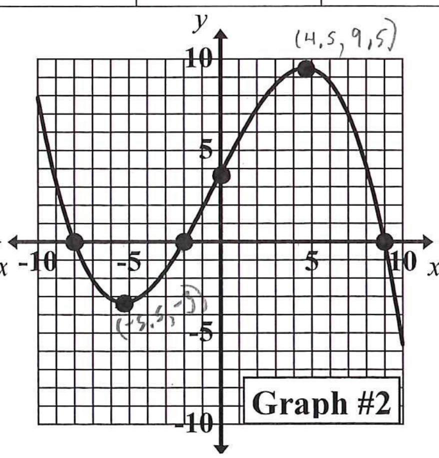
Name ______________________________ Period __________

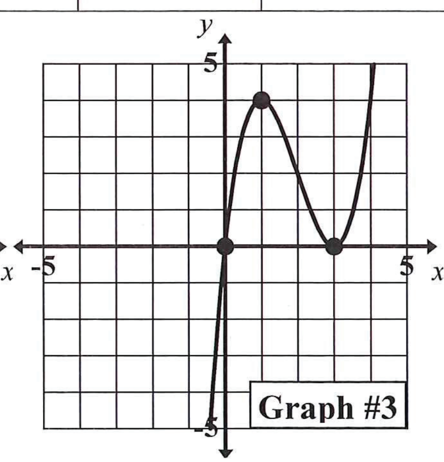
6.1A *Graphing Cubic Functions: Significant Features*

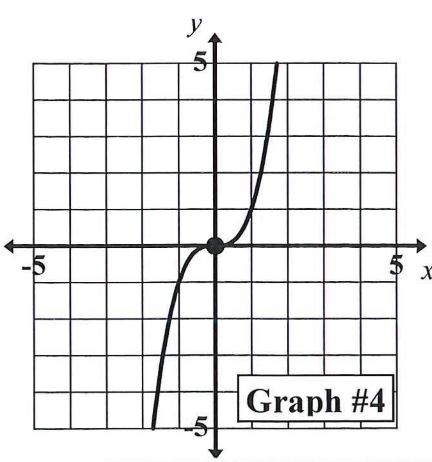
7. Determine which graph below has the identified value as a significant feature. Then use the graph to complete the table.

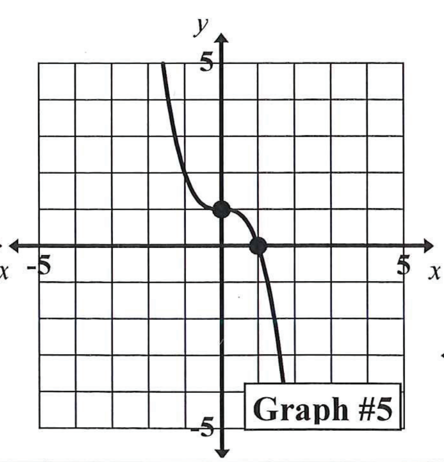
Graph #:	1	3	5	4	6	2
x-intercept(s):	$(-5,0)$ $(3,0)$ $(7,0)$	$(0,0)$ $(3,0)$	$(1,0)$	$(0,0)$	$(-2.5,0)$	$(-8,0)$ $(-2,0)$ $(9,0)$
y-intercept(s):	$(0,-5.5)$	$(0,0)$	$(0,1)$	$(0,0)$	$(0,4)$	$(0,4)$
Relative Maximum:	$(5,2)$	$(1,4)$	none	none	$(-1,7)$	$(4.5,9.5)$
Relative Minimum:	$(-2,-7)$	$(3,0)$	none	none	$(1.1,0.9)$	$(-5.5,-3.5)$
interval(s) where function values are increasing:	$-2<x<5$	$x<1$, $x>3$	none	Increasing over entire domain	$x<-1$, $x>1.1$	$-5.5<x<4.5$
interval(s) where function values are decreasing:	$x<-2$, $x>5$	$1<x<3$	Decreasing over entire domain	none	$-1<x<1.1$	$x<-5.5$, $x>4.5$

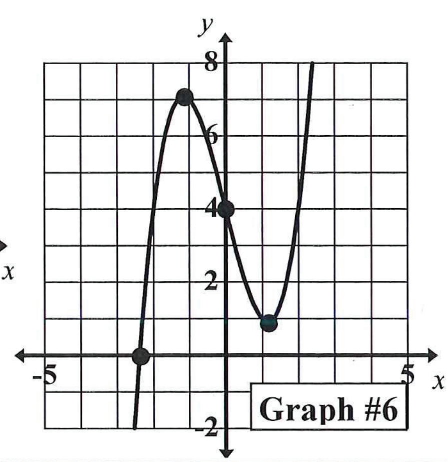












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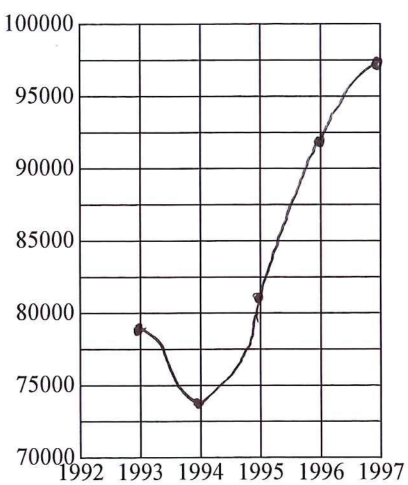
6.1A *Graphing Cubic Functions: Significant Features*

8. Draw a graph for the sets of data as reported in the USDA Statistical Highlights. From the graph, determine whether the data could be modeled with a cubic function. Give reasons for your answers.

a) Number of acres of fresh carrots harvested in the United States.

Year	Acres
1993	78,220
1994	74,630
1995	81,120
1996	92,160
1997	97,460

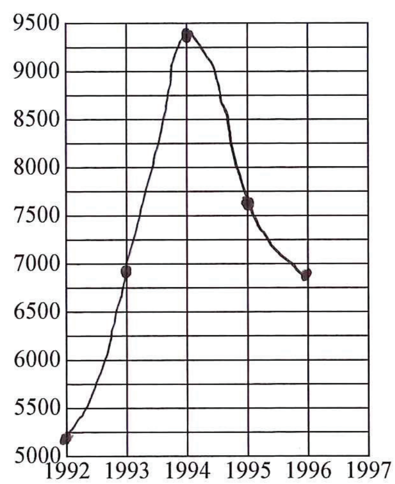


Possible cubic function? yes

If before year 1993, the # acres was < 74,630 and declining each previous year, then the end beh would be ↙↗.

b) Number of bales of cotton (in thousands) exported.

Year	Acres
1992	5,200
1993	6,860
1994	9,400
1995	7,680
1996	6,870

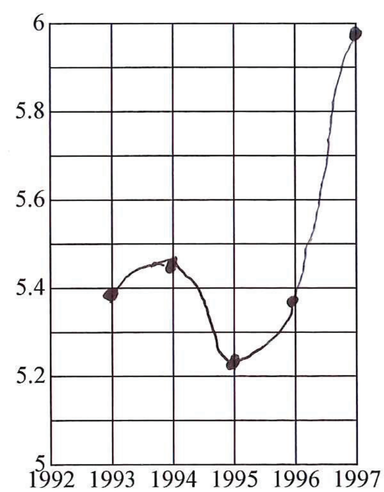


Possible cubic function? yes

If sometime after 1996 the #bales was > 9400 thousand and increasing each succeeding year, the end beh would then be ↙↗.

c) Yield per acre (in thousands) for processed cucumbers.

Year	Acres
1993	5.38
1994	5.44
1995	5.22
1996	5.37
1997	5.98



Possible cubic function? yes

If before the year 1993, the yield/acre was < 5.22 thousand. and decreasing each previous year, the end beh would then be ↙↗.

9. Write a cubic function that has the following end behavior: (↙, ↗)

one ex: $f(x) = x^3 + x + 1$

10. Write a cubic function that has the following end behavior: (↖, ↘)

one ex: $g(x) = -2x^3 + 3x^2 - x + 4$

Section 6.1A